CBCS SCHEME

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17MAT11

First Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the n^{th} derivative of $\sin 2x \cos x$. **(06 Marks)**

b. Prove that the following curves cuts orthogonally $r = a(1+\sin\theta)$ and $r = a(1-\sin\theta)$. **(07 Marks)**

c. Find the radius of the curvature of the curve $r = a\sin n\theta$ at the pole. **(07 Marks)**

OR

2 a. If $\tan y = x$, prove that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. **(06 Marks)**

b. With usual notations, prove that $\tan\phi = \frac{\tau d\theta}{dr}$. **(07 Marks)**

c. Find the radius of curvature for the curve $n^2 y = a(x^2 + y^2)$ at (–2a, 2a). **(07 Marks)**

Module-2

3 a. Using Maclaurin's series prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \ldots$ **(06 Marks)**

b. If $U = \cot^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = -\frac{1}{4}\sin 2U$. **(07 Marks)**

c. Find the Jacobian of $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$. **(07 Marks)**

OR

4 a. Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x}$. **(06 Marks)**

b. Find the Taylor's sense of log(cos x) about the point $x = \frac{\pi}{3}$ upto the third degree. **(07 Marks)**

c. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. **(07 Marks)**

Module-3

5 a. If $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ represents the parametric equation of a curve then, find velocity and acceleration at t = 1. **(06 Marks)**

b. Find the constants a and b such that $\vec{F} = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Prove that $\text{div}(\text{curl}\,\vec{A}) = 0$. **(07 Marks)**

OR

6 a. Find the component of velocity and acceleration for the curve $\vec{r} = 2t^2 i + (t^2 - 4t)j + (3t - 5)k$ at the points t = 1 in the direction of $i - 3j + 2k$. **(06 Marks)**

b. If $\vec{t} = \nabla(xy^3z^2)$, find div $\vec{t}$ and curl $\vec{t}$ at the point (1, −1, 1). **(07 Marks)**

c. Prove that curl(grad ϕ) = 0. **(07 Marks)**

Module-4

7 a. Prove that $\int_0^2 \frac{x^4}{\sqrt{4 - x^2}} dx = 3\pi$ using reduction formula. **(06 Marks)**

b. Solve $(x^2 + y^2 + x)dx + xydy = 0$. **(07 Marks)**

c. Find the orthogonal trajectory of $r^n = a \sin n\theta$. **(07 Marks)**

OR

8 a. Find the reduction formula for $\int \cos^n xdy$ and hence evaluate $\int_0^{\pi/2} \cos^n xdx$. **(06 Marks)**

b. Solve $ye^{xy}dx + (xe^{xy} + 2y)dy = 0$. **(07 Marks)**

c. A body in air at 25°C cools from 100°C to 75°C in 1 minute. Find the temperature of the body at the end of 3 minutes. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by reducing to row echelon form. **(06 Marks)**

b. Find the largest eigen and the corresponding eigen vector for $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by taking the initial approximation as $[1, 0.8, -0.8]^T$ by using power method. Carry out four iterations. **(07 Marks)**

c. Show that the transformation $y_1 = 2x_1 - 2x_2 - x_3$, $y_2 = -4x_1 + 5x_2 + 3x_3$, $y_3 = x_1 - x_2 - x_3$ is regular. Find the inverse transformation. **(07 Marks)**

OR

10 a. Solve the equations $5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$ by using Gauss Seidal method. Carryout three iterations taking the initial approximation to the solution as (1, 0, 3). **(06 Marks)**

b. Diagonalize the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. **(07 Marks)**

c. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form by orthogonal transformation. **(07 Marks)**

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